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Anomalous diffusion on regular and random models for diffusion-limited aggregation

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Abstract. Families of regular fractal models for Witten-Sander (DLA) clusters in d dimensions are proposed resembling the computer generated aggregates. We evaluate random walk dimensionalities exactly using resistance scaling ideas. The applicability of the conflicting conjectures of Alexander-Orbach (AO), $d_w = 3d_f/2$, and Aharony-Stauffer (AS), $d_w = d_f + 1$, to DLA is then examined. We find that the dendritic nature of the fractals presented ensures that AS is satisfied exactly in all dimensions. An RSRG fugacity transformation method is applied to the simplest of the regular fractals in $d = 2$. The result for d_w is compared with the exact resistance scaling result to obtain an estimate of the error introduced by the RSRG. The RG method is then generalised to treat the random DLA problem. The resulting estimate for d_w is in fortuitously good agreement with AS, the corresponding Monte Carlo data and the results above for the regular fractal models.

1. Introduction

The problem of random walks on percolation clusters at the percolation threshold (Rammal and Toulouse 1983, Gefen *et al* 1983), lattice animals (Family 1983, Gould and Kohin 1984) and other such 'fractal' systems, e.g. hierarchical lattices (Given and Mandelbrot 1983), has received much attention since it was first introduced by de Gennes (1976) in the context of the 'ant in the labyrinth'.

Whereas such random fractals as percolation clusters or lattice animals are equilibrium clusters, DLA clusters (Witten and Sander 1981, 1983) are formed kinetically and it is found that their static 'critical' behaviour lies in a new universality class (Gould *et al* 1983). Much less is known about random walks on DLA (Meakin and Stanley 1983) and in particular (Guyer 1984) whether spectral and walk dimensions of DLA satisfy conjectured relationships to the fractal dimension. This topic is studied in the present paper.

The growth process of a DLA cluster, as described by Witten and Sander (1981), is as follows. At $t = 1$ a seed particle is placed at the centre of a large hypersphere. At $t = 2$ a second particle is released at a random point on the hypersphere and allowed to perform a random walk until it reaches a perimeter site, where it sticks. At $t = 3$ a third particle is released and so on until a large aggregate is built. The DLA aggregate so formed has an anomalous scale invariance. For instance, the density-density correlation function $\langle \rho(r'+r)\rho(r')\rangle$ has the following asymptotic behaviour:

$$\langle \rho(r'+r)\rho(r')\rangle \sim r^{-A}. \tag{1.1}$$

The exponent A is related to the fractal (or Hausdorff) dimension (Mandelbrot 1982) d_f of the aggregate, which is defined by the following relationship between linear dimension ξ_f and number of particles N_f (both large) of the cluster:

$$N_f \sim (\xi_f)^{d_f}. \tag{1.2}$$

A volume integration of (1.1) leads to the relation

$$d_f = d - A \tag{1.3}$$

where d is the space dimension.

The statistical scale invariance underlying (1.1) and (1.2) makes the theory of DLA expressible in the language of critical phenomena.

As a consequence of the specific growth process involved, DLA aggregates are distinctly different fractals (having different values of d_f) from either percolation clusters or random lattice animals (Gould *et al* 1983). It then also becomes necessary to understand the relevance of the specific growth type to dynamic processes such as diffusion on the DLA.

Apart from intrinsic interest in DLA as an example of a fractal in a new universality class it has applications to a variety of systems. DLA has been used as a model for growth of tumours (Williams and Bjerknes 1972), coagulation of smoke particles (Forrest and Witten 1979), growth of crystals from an undercooled melt or a supersaturated solution (Langer 1980), turbulence (Hentschel and Procaccia 1982), dielectric breakdown (Niemeyer *et al* 1984) and viscous fingering (Nittmann *et al* 1985).

Now, in classical diffusion in Euclidean space only a single length scale exists. This length is the RMS displacement ξ_w of the random walker from the origin. The number of steps N_w of the walk is related to ξ_w in the limit $N_w \gg 1$ by a power law form from which one defines the dimensionality of the walk d_w:

$$N_w \sim (\xi_w)^{d_w}. \tag{1.4}$$

Diffusion processes on such Euclidean spaces are characterised by the Flory result $d_w = 1/\nu = 2$ valid for all d.

When the 'ant' is on a fractal a second competing length occurs. In the case of DLA this length is the radius of gyration ξ_f, introduced above as a measure of the linear dimension of the fractal cluster.

For diffusion on such fractal systems ($d_f < d$) we expect a crossover between the asymptotic behaviours of two limiting regimes. In the first one $\xi_w \gg \xi_f$ and $d_w = 2$. We are interested in the 'self-similar' regime where $1 \ll \xi_w \ll \xi_f$. In this limit d_w depends on the particular system and the diffusion is said to be anomalous.

Further, Alexander and Orbach (1982) observed that the anomalous scaling of the density of states on a percolation cluster was governed by an exponent, the spectral or fracton dimension d_s, which appeared superuniversal in that it was independent of d. Alexander and Orbach showed that $d_s = 2d_f/d_w$ and conjectured that

$$d_s (= 2d_f/d_w) = \tfrac{4}{3} \tag{1.5}$$

independently of d. The AO conjecture has been extended to apply to homogeneous fractals (see Leyvraz and Stanley 1983) of which DLA clusters are supposed to be an example.

However, recent arguments against the AO conjecture have emerged (see Family and Coniglio 1984) and accurate numerical work (Zabolitzky 1984) seems to indicate that AO fails in $d = 2$. Aharony and Stauffer (1984) have presented an argument that applies to fractals with $d_f \leqslant 2$ and they conclude that

$$d_w = d_f + 1 \tag{1.6}$$

implying a breakdown of the AO rule $d_w = 3d_f/2$. Specifically, the AS conjecture should apply to DLA in two dimensions.

In the present paper we test the AO and AS conjectures for simple and generalised regular fractal models of DLA. We then proceed to apply RSRG ideas to the random system. Our results favour the latter conjecture.

The plan of the paper is as follows. In § 2 our initial investigation of the simplest of regular models for DLA, originally proposed by Vicsek (1983), is similar to the work of Guyer (1984) who treated walks on the fractal using RG on a diffusion equation. We map the system trivially onto an equivalent resistor network and use the method of resistance scaling to derive the analogue of the percolation conductivity exponent t and hence the dimensionality of walks on the regular fractal. We extend the method to the corresponding fractal family in d dimensions. We repeat the method for another, possibly more authentic, regular fractal family in d dimensions. Finally we investigate the effect of introducing loops on the spectral dimensionality.

In § 3 we limit ourselves to $d = 2$ and apply real space renormalisation group fugacity transformation methods to the simpler of the fractals mentioned above. Results for d_w are obtained and compared with the exact results from § 2. This serves as a check for the RSRG method and indicates whether one is likely to obtain reasonable results when one applies it to the random case.

In § 4 we treat DLA proper (the random case) using RSRG and compare our results with those of the corresponding Monte Carlo simulation (Meakin and Stanley 1983).

2. Resistance scaling for regular models of diffusion-limited aggregates

When proposing suitable regular models for DLA aggregates, one must preserve the essential physical attributes of the random structures as suggested by Monte Carlo pictures (see Meakin 1983). For instance, the models should have some form of rotational symmetry and have suitable fractal dimensionality. They must also be dendritic (i.e. have no loops) as this seems to be a characteristic feature of the aggregates.

In dealing with DLA we are *ipso facto* dealing with site structures. In order to introduce a resistor network, it is however necessary to express the problem in terms of resistors or 'bonds'. We therefore need a non-random model of a DLA cluster which builds the cluster hierarchically but is composed of both sites and bonds. In figure 1 we show three successive stages in the iterative construction of the simplest of such models (Vicsek 1983). At each iteration the lattice spacing a is reduced to, say, a/b and more detail of the fractal is generated. This process is repeated such that $a \to a/b \to a/b^2 \to \ldots \to a/b^n$. In the limit $n \to \infty$ we obtain the regular fractal shape.

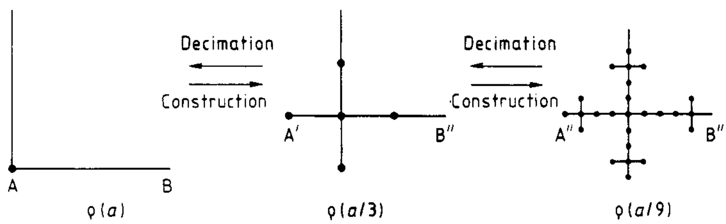

Figure 1. Recursive construction of regular fractal resistor network for DLA composed of sites and bonds.

This construction may be reversed by decimation. Here short range degrees of freedom are eliminated followed by a length scale dilatation $a \to ba$. However, if we now assign a resistance $\rho(a)$ to each bond and, in the decimation-dilatation process, conserve the potential difference between two sites separated by a fixed length ξ, we then have the following scaling ansatz for the resistance:

$$\rho(ba) = b^{d_r}\rho(a). \tag{2.1}$$

b is here the length scale dilatation factor and d_r is the fractal dimension of the effective one-dimensional resistance length of the aggregate. We then use the relation

$$d_w = d_f + d_r \tag{2.2}$$

due to Alexander and Orbach (1982) and Stanley and Coniglio (1984) to determine the random walk dimensionality d_w.

We illustrate the method for a $b = 3$ dilatation and perform the decimation indicated in figure 2(a). Let V_{AB} be the potential difference between A and B when current I flows from A subject to $V_{AB} = V_{AC} = V_{AD}$. By trivial application of Ohm's and Kirchoff's laws one finds that

$$V_{AB}(a) = \tfrac{3}{2}I\rho(a) \qquad V_{A'B'}(ba) = \tfrac{1}{2}I\rho(ba). \tag{2.3}$$

It then follows using the scaling ansatz that

$$d_r = \log 3/\log 3 = 1 \tag{2.4}$$

so using $d_w = d_f + d_r$ we obtain

$$d_w = \log 15/\log 3 \approx 2.47. \tag{2.5}$$

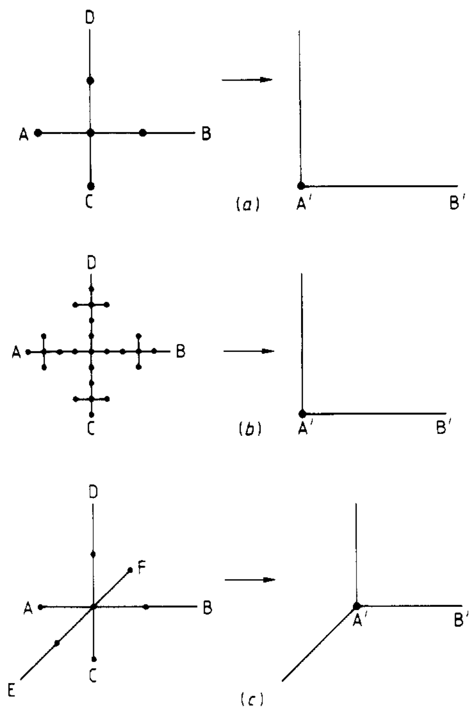

Figure 2. Decimation for simple fractal model with length scale dilatation factor (a) $b = 3$ and (b) $b = 9$. (c) Decimation for simple model in three dimensions with scale factor $b = 3$.

Repeating the process for a $b=9$ dilatation (figure 2(b)) one obtains

$$d_r = \log \tfrac{144}{13}/\log 9 \tag{2.6}$$

and so

$$d_w = \log \tfrac{3600}{13}/\log 9 \approx 2.56. \tag{2.7}$$

In fact the calculation is easily done for general $b=3^N$ and yields

$$d_r = \log[(b-1)+(b^2-1)/(3b-1)]/\log b. \tag{2.8}$$

In the limit of $b \to \infty$ we recover

$$d_r = 1. \tag{2.9}$$

This result agrees exactly with the AS conjecture. It implies that the path followed by a particle, in traversing a region of the fractal of linear extent of order ξ_f, is essentially one dimensional, albeit highly decorated. We therefore obtain for the random walk dimensionality d_w

$$d_w = \log 15/\log 3 \approx 2.47 \tag{2.10}$$

and for the spectral dimensionality d_s

$$d_s = 2d_f/d_w = \log 25/\log 15 \approx 1.19. \tag{2.11}$$

The result above for d_s is over 10% lower than the AO prediction but in exact agreement with the AS conjecture. It is also consistent with the work by Guyer (1984) on the same model who used decimation on a discrete diffusion equation and obtained $d_r = 1$. The result also agrees well with the Monte Carlo simulation of Meakin and Stanley (1983) on the random system which gave $d_s = 1.20$ (+0.10, −0.05).

It is straightforward to generalise the above to apply for arbitrary embedding dimension d. We construct a family of fractals such that the 'unit cell' is now a d-dimensional cross with $2d$ arms (the case $d=3$ is shown in figure 2(c)). From this one obtains

$$d_f = \log(2d+1)/\log 3. \tag{2.12}$$

The resistance rescaling calculation in general d again yields the AS result $d_r \to 1$ as $b \to \infty$. One therefore has

$$d_w = \frac{\log 3(2d+1)}{\log 3} \qquad d_s = \frac{2\log(2d+1)}{\log 3(2d+1)}. \tag{2.13}$$

In particular then for $d=3$, we have $d_s \approx 1.28$ which again compares favourably with the corresponding Monte Carlo result (Meakin and Stanley 1983) for the random system $d_s = 1.30 \pm 0.06$.

From (2.12) it is evident that for $d>4$ we have $d-d_f>2$. This however makes the cluster transparent to incoming random walkers whose path has dimensionality $d_w=2$. The incoming particles would therefore enter into the central regions of the aggregate and alter its structure, increasing its fractal dimension d_f. It would appear then that the family of fractals presented here are poor models for DLA aggregates for $d>4$. However, the above problem is inherent in all such families of regular fractal representations of DLA clusters and arises here from the logarithmic dependence of d_f on d.

The above is repeated for the fractal shown in figure 3. This shape is more realistic than the previous one, particularly in $d=2$ since its fractal dimension $d_f(d=2)$ is ($\log 73/\log 13$) ≈ 1.673 in very good agreement with the corresponding Monte Carlo

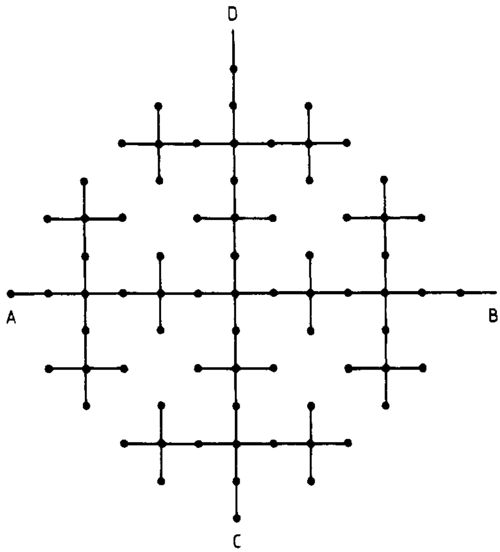

Figure 3. Alternative model for DLA. In two dimensions this model has $d_f = \log 73/\log 13 \approx 1.673$ in very good agreement with numerical estimates for the random case. The dendritic structure of Monte Carlo generated pictures has been preserved.

result $d_f = 1.678 \pm 0.047$ (Meakin 1983). In general d the $b \to \infty$ resistance rescaling results for this family are

$$\begin{aligned} d_f &= \log(1+4d+8d^3)/\log 13 \\ d_w &= \log[13(1+4d+8d^3)]/\log 13 \\ d_s &= 2\log(1+4d+8d^3)/\log 13(1+4d+8d^3). \end{aligned} \tag{2.14}$$

The AO conjecture once again is violated whereas AS holds exactly. Notice that this family encounters the same problems as the former one for $d > 4$.

Although we have limited our attention above to models with a C_4 symmetry the result $d_r = 1$ follows for dendritic fractals with general C_n symmetry.

Finally, we introduce loops into our model and note their impact on diffusion on the cluster. We perform a $b = 5$ rescaling on the model of figure 4. Solving the resulting set of eleven simultaneous equations yields $d_r = 1.032$ and so $d_s = 1.214$ while the analogous $b = 5$ calculation on the same model without the loops gives $d_r = \log\frac{40}{7}/\log 5 \approx 1.083$ and so $d_s = 1.115$. This 10% fall in the spectral dimensionality

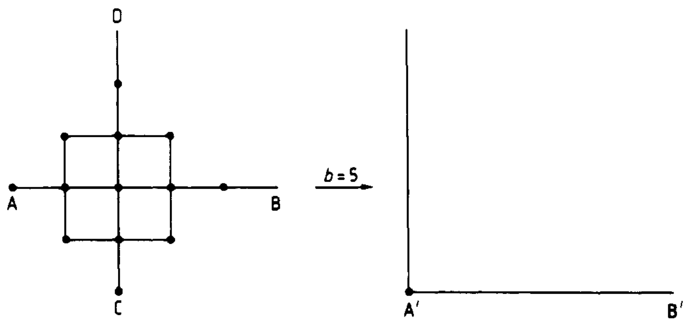

Figure 4. Decimation for a regular fractal, having loops on all length scales, by a factor $b = 5$.

d_s on elimination of the loops in models in $d=2$ is typical and itself accounts for the deviation from the AO conjecture observed above in the $b \to \infty$ limit.

In the following section we use the regular models for DLA introduced above as fractal lattices upon which to apply fugacity transformation methods (Stanley *et al* 1982). This procedure then gives an error bound for the RSRG which would otherwise contain an uncontrolled approximation. We anticipate that the error introduced by the RSRG on the random system is similar to that introduced on the regular model.

3. RSRG approach to random walks on regular models of diffusion-limited aggregates

In forming the recursion relations below, we use the kinetic interpretation (Nakanishi and Family 1984) for the random walks as the local coordination number varies from site to site.

In evaluating each transformation our 'ant' begins his walk at A and finishes at B. For a walk of length n we assign a weight $W^n\{(1/Z_0)(1/Z_1)\ldots(1/Z_{n-1})\}$ where Z_i is the local coordination number of the site that the ant has reached after i steps. Finally we sum over all possible walks to form the generating function $G(W)$. We associate the renormalised fugacity W' with this generating function through

$$\tfrac{1}{2}W' = \sum_{\text{walks}} W^n\{(1/Z_0)(1/Z_1)\ldots(1/Z_{n-1})\}. \tag{3.1}$$

This recursion relation has a critical unstable fixed point at $0 < W^* < \infty$ and if we define $\lambda_w = (\partial W'/\partial W)_{w^*}$ we obtain

$$d_w = \log \lambda_w / \log b. \tag{3.2}$$

For computational reasons the infinite series $G(W)$ must be truncated at Nth order in W. This has the effect of ignoring walks with number of steps n greater than N. One problem is to decide when to truncate.

The diffusion is on a fractal space with $d_f < d$ and as mentioned in the introduction two length scales ξ_w, ξ_f exist. We are interested in the self-similar limit and so we require that the maximum number of steps in a walk be restricted to $\xi_w \ll \xi_f$. At the critical fugacity W^* only random walks of length $\xi_w \sim N^{(1/d_w)}$ are important. We therefore ignore walks with $N > \xi_w^y$, with $y = d_w$. However, we have no *a priori* knowledge about d_w except that $d_s > 1$ and $d_w > d_f$ and these imply that

$$d_f < d_w < 2d_f. \tag{3.3}$$

We therefore (cf Sahimi and Jerauld 1984) only treat walks whose number of steps N satisfy

$$N \leqslant (\xi_w)^{3d_f/2}. \tag{3.4}$$

For example, for the model of figure 1 we choose $y \approx 2.197$. ξ_w is the end-to-end length of the random walk, which in the models we investigate equals the dilatation factor b. The legitimacy of the procedure was checked by using shorter or longer walks and little change in the results for d_w was observed, for example, inserting $y \in [2.0, 2.4]$ changes the extrapolated results by at most 4%.

The explicit summation over all possible random walks was carried out using a method described in Family and Gould (1984).

The results obtained (for the model of figure 1) were: $d_w = 1.73$ from a $b=3$ renormalisation; $d_w = 1.89$ (for $b=9$); $d_w = 1.97$ (for $b=27$). We now assume (Reynolds *et al* 1980) that $d_w(b)$ is a quadratic in $(\log b)^{-1}$:

$$d_w(b) = d_w + C_1/(\log b) + C_2/(\log b)^2. \tag{3.5}$$

On extrapolating we find that in the $b\to\infty$ limit, $d_w = 2.14$ which yields for the spectral dimension $d_s = 1.37$. This result is in qualitative agreement with the Monte Carlo result, for the random case, of Meakin and Stanley ($d_s = 1.20$ (+0.10, −0.05)) and the resistance rescaling technique for the same model ($d_s = 1.19$).

In the light of these results we proceed below with a related RSRG for the random DLA system.

4. An RSRG for random walks on DLA

We now present a single cell RSRG for deriving d_w for random DLA in two dimensions. This involves three fugacity parameters. A fugacity K is associated with each occupied site of the cell, another fugacity W is associated with each step of the incoming particles and a third fugacity V is associated with each step of the random walker on the cluster. The calculation is in two stages. Firstly one constructs all possible spanning clusters in a finite $b\times b$ cell. Universality arguments ensure that no generality is lost by limiting the treatment to a square lattice. One begins with a seed site in the lower left-hand corner of the cell and introduces a random walker from the North or East. The random walker then diffuses through the cell and having landed on a site adjacent to the seed site it remains there and a new random walker is introduced and the process repeats itself. One then evaluates the number, C_{st}, of ways of growing a spanning cluster of s sites generated by random walks with total number of steps t. To preserve the symmetry we define a cluster to be spanning if it can be traversed both vertically and horizontally. The RSRG transformation for K is then given by (Gould *et al* 1983)

$$K' = \sum_{s,t} C_{st} K^s W^t. \tag{4.1}$$

This is analogous to the RSRG for lattice animals (Family 1983) and self-avoiding walks (de Queiroz and Chaves 1980).

To derive the recursion relation for W one renormalises all the possible walks on a $b\times b$ cell to a single step on the renormalised lattice as indicated in figure 5(a). As the incoming particles are diffusing on a pure lattice we may use the static interpretation for the random walks (see Nakanishi and Family 1984). One then obtains

$$W' = \sum_n C_n W^n \tag{4.2}$$

C_n being the number of random walks of n steps spanning the cell in one direction. This transformation has two stable fixed points at $W=0$, $W=\infty$ and an unstable fixed point in $0<W^*<\infty$. One then proceeds to find the doubly unstable fixed point (W^*, K^*) of the pair of transformations and then d_f follows in the usual way from the eigenvalue of the linearised transformation (4.1) at this fixed point. Gould *et al* (1983) find $d_f = 1.71$ for $b=2$ and $d_f = 1.67$ for $b=3$.

The next stage is to consider random walks on the DLA aggregate. As in § 3 we use the kinetic interpretation for the walks on the aggregate. We generate all possible spanning clusters in all possible ways as above and for each one we evaluate all the random walks on the cluster that start from the lower-left corner and traverse the cell vertically. Associating the generating function so derived with the corresponding

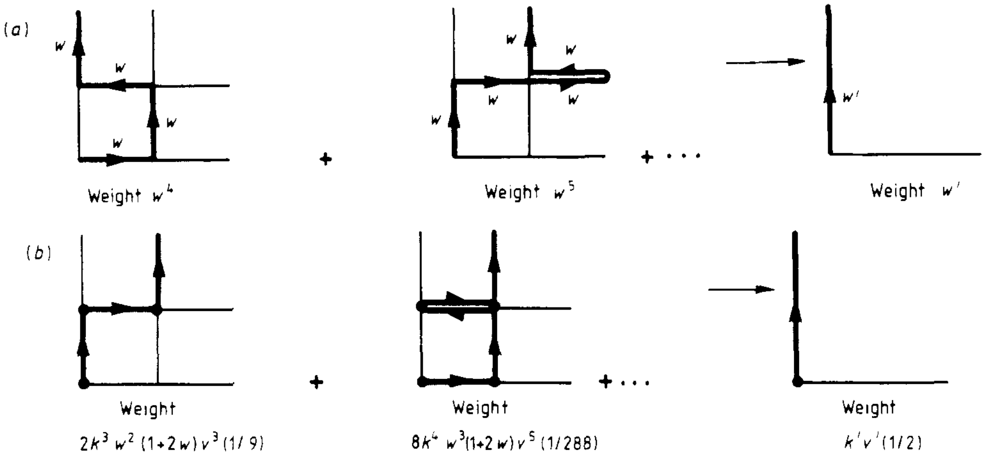

Figure 5. (a) $b=2$ static renormalisation for random walks on a pure lattice. Each step has weight W. All walks, originating from the lower-left corner, traversing the $b \times b$ cell vertically with number of steps $N \leqslant b^{1/d_w}$ are weighted, summed and renormalised to W', the single step fugacity on the renormalised lattice. (b) $b=2$ kinetic renormalisation for random walks on DLA. Each step of the random walk has weight V/Z_a for local coordination number Z_a. All spanning random walks on all spanning cluster configurations are weighted appropriately, summed and renormalised to $\frac{1}{2}K'V'$.

quantity on the renormalised lattice, one obtains

$$\tfrac{1}{2}K'V' = \sum_{s,n} B_{sn}\left(\sum_{t} C_{st} K^s W^t\right) V^n \tag{4.3}$$

where B_{sn} is the number of walks of n steps spanning in one direction in a cluster of s sites (together with a factor associated with the kinetic interpretation of the random walk on the cluster).

One then evaluates the critical fixed point $0 < V^* < \infty$ and the associated eigenvalue $\lambda_v = (\partial V'/\partial V)_{K^*,W^*,V^*}$ from which one derives $d_w = \log \lambda_v / \log b$. We perform the procedure explicitly for a $b=2$ cell. As shown by Gould *et al* (1983), the recursion relations for K and W are

$$K' = 6K^3W^2(1+2W) + 8K^4W^3(1+2W) \qquad W' = W^2 + 2W^3 + 5W^4 + 14W^5 \tag{4.4}$$

and we obtain for V'

$$\tfrac{1}{2}K'V' = (\tfrac{7}{12}\alpha + \tfrac{1}{6}\beta)V^2 + (\tfrac{2}{9}\alpha + \tfrac{1}{12}\beta)V^3 + (\tfrac{55}{108}\alpha + \tfrac{5}{36}\beta)V^4 + (\tfrac{26}{81}\alpha + \tfrac{1}{9}\beta)V^5 \tag{4.5}$$

with $\alpha = 2W^2(1+2W)K^3$ and $\beta = 8W^3(1+2W)K^4$. The critical fixed point is

$$\begin{bmatrix} K^* \\ W^* \\ V^* \end{bmatrix} = \begin{bmatrix} 0.766 \\ 0.347 \\ 0.984 \end{bmatrix} \tag{4.6}$$

and the eigenvalues here yield $d_f = 1.71$ and $d_w = 1.74$. The spectral dimensionality is then $d_s = 1.97$.

For the $b=3$ cell we used a FORTRAN program on a VAX 11/780 machine to evaluate the transformations for K, W and V and hence the dimensionalities:

$$d_f = 1.67 \qquad d_w = 1.98. \tag{4.7}$$

The closed form calculations for larger cells were not feasible owing to the enormous amount of computer time needed. We cannot therefore exploit the usual extrapolation scheme (3.5) but as a first approximation to the $b \to \infty$ limit of d_w we perform linear extrapolation in $(\log b)^{-1}$, i.e. we assume that d_w takes the form $d_w(b) = d_w + C(\log b)^{-1}$. This yields $d_w = 2.39$ and $d_s = 1.39$. The error associated with the RSRG on the regular system only affects d_w. Similarly, for the random system d_f is in excellent agreement with the numerical data. It is therefore reasonable to expect that the errors in the RG treatment of the regular and random DLA models may be related; we assume the relationship (which is supported, see the appendix, by a test on percolation infinite cluster models)

$$\frac{d_s(\text{exact, random})}{d_s(\text{RG, random})} \approx \frac{d_s(\text{exact, regular})}{d_s(\text{RG, regular})}. \tag{4.8}$$

Inserting our results into (4.8) yields d_s (exact, random) $\approx (1.19/1.37) \times 1.39 \approx 1.21$ which is fortuitously close to that observed in the Monte Carlo work of Meakin and Stanley (1983) $d_s(d=2) = 1.20\ (+0.10, -0.05)$.

5. Discussion

The aims of this paper have been to establish which structural features of DLA aggregates are relevant to diffusion processes and to evaluate the spectral dimensionality d_s as a possible test of the conflicting conjectures of Alexander-Orbach and Aharony-Stauffer for these systems.

We have proposed regular fractal models on which calculations could be carried out exactly. Models were chosen suggestive of the Monte Carlo generated random aggregates. We note that the models used are neither homogeneous nor random and therefore violate the hypotheses of the AO and AS conjectures respectively. However, their structural similarity to their random counterparts indicates that results obtained for these regular models are indeed relevant to DLA. This seems to be borne out by the results of § 4 and the existing Monte Carlo data.

We find that the most important characteristic of the models as far as diffusion is concerned is that they are *dendritic*. This fact is sufficient to account for the discrepancy between the value of d_s from the numerical work of Meakin and Stanley (1983) and the AO estimate of $\frac{4}{3}$ in two dimensions. Other considerations such as the nature of rotational symmetry of the aggregates or actual fractal dimension seem secondary. This would suggest, for example, that as one decreases the sticking probability p in the generation of the aggregate, since d_f appears to be unchanged (Meakin 1983), i.e. p seems to be an irrelevant variable, and the structure remains dendritic, then the spectral dimension d_s will also remain essentially unchanged. Furthermore, consider the aggregate to be constructed from particles whose motion is characterised not by random walks but having instead Levy flight trajectories such that

$$P(x \geqslant U) = U^{-f} \qquad P(x<1) = 0 \tag{5.1}$$

where $P(x \geqslant U)$ is the probability that the length of the step will be greater than or equal to U. The numerical work of Meakin (1984) indicates that d_f *is* affected, i.e. f is a *relevant* variable, while the dendritic nature of the clusters is preserved. Our reasoning suggests therefore that the dynamic properties of these new aggregates are determined uniquely in terms of the modified static geometry, in particular $d_s = 2d_f/(d_f+1)$. Monte Carlo work needs to be done to confirm this.

We have shown that the resistance scaling exponent d_r is equal to unity in our regular models for all values of the embedding dimension d; this is another consequence of their dendritic nature. Our results are in agreement with the related work of Guyer (1984) on the regular Vicsek model. The value $d_r = 1$ obtained for the DLA fractal models is consistent with the AS conjecture for $d = 2$, since there $d_f \leq 2$. This feature persists in the real clusters and so, if it is the only relevant feature, the AO conjecture will fail for DLA while the AS conjecture will hold exactly not only for d such that $d_f \leq 2$ but in general embedding dimensions. Further, Havlin (1984) has suggested that on a treelike structure the spectral dimension should be given by $d_s = (2d_l/d_l + 1)$, where d_l is the 'chemical' dimension. For DLA, it seems that $d_l = d_f$, since the chemical distance scales linearly with Euclidean distance. Hence the AS rule should apply, as indeed is found here.

Finally, in § 4, we obtained an estimate for d_s in the random system using RSRG and the results of § 3. The result obtained is in good agreement with the numerical data, supporting the conclusion that $d_s(d = 2)$ falls well short of the AO prediction for DLA clusters.

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Appendix

We test the legitimacy of (4.8) by applying it to a related system, percolation. In the spirit of § 3 we have used fugacity transformation methods to determine d_s for Sierpinski gasket models of the infinite incipient cluster in two dimensions. Strictly speaking, Sierpinski gaskets model the IIC backbone but here we use them as simple models for the full infinite cluster. Explicitly, we have performed $b = 2, 4, 8$ renormalisations as in figure 6 and obtained $d_w(2) = 1.82$, $d_w(4) = 2.01$, $d_w(8) = 2.08$ using the midpoint rule as above. We then apply the extrapolation rule $d_w(b) = d_w + C_1/(\log b) + C_2/(\log b)^2$ to obtain $d_w = 2.22$ and hence $d_s = 1.43$. Gefen *et al* (1981) have performed the resistance rescaling calculation for the Sierpinski gasket and obtain

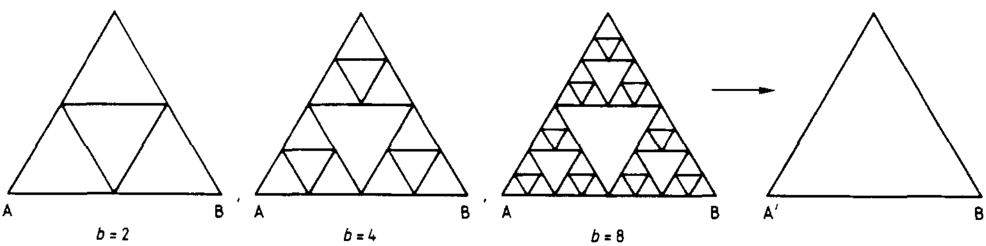

Figure 6. $b = 2, 4, 8$ decimation for the Sierpinski gasket model of the percolation IIC. During the resistance rescaling $V_{AB} = V_{A'B'}$. For the RG process, random walks spanning from A to B are weighted and renormalised to $\frac{1}{2}V'$, the weight of the renormalised step from A′ to B′.

$d_s = 2\log 3/\log 5$ and therefore, for percolation in $d=2$, the RHS of (3.13) becomes

$$\frac{d_s(\text{exact, regular})}{d_s(\text{RG, regular})} \approx 0.954. \tag{A1}$$

Now, for the random system we use the accurate numerical results of Zabolitzky (1984) (d_s('exact', random) = 1.322) and the RG results of Reynolds *et al* (1980), $d_f = 1.898$ and Sahimi and Jerauld (1983), $d_w = 2.810$ (yielding the estimate d_s(RG, random) = 1.351). The LHS of (4.8) thus becomes

$$\frac{d_s(\text{exact, random})}{d_s(\text{RG, random})} \approx 0.979. \tag{A2}$$

The error involved in using the relation (4.8) for percolation is therefore less than 2.5%. This result therefore supports the argument given at the end of § 4 for the use of (4.8).

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